

Hemispherical Projection of the Four-Dimensional Sphere for Continuous Gesture-Based Weighted Selection

*A Mathematical Formalization of the 3D Hemispherical Selector with Rotation Swipes and Zoom for Accurate
Continuous Vote $C = X = Z$*

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ABSTRACT

This monograph formalizes an original interface concept in which the stereographic projection of a 3-sphere embedded in four-dimensional Euclidean space yields a three-dimensional hemispherical selector. Rotation and swipe gestures map to continuous angular parameters that distinguish approval from disapproval, while pinch and spread gestures control the volumetric weight of the selection. The continuous coordinates extracted from the projection supply the pair (X, Z) that defines the cast vote by the identification $C = X = Z$, producing a high-precision continuous choice rather than a discrete ballot. Foundational geometry of the 3-sphere, stereographic projection, spherical volume computations, and mid-air gesture literature are reviewed and then specialized to the selector model. Refined analysis incorporates primary sources from Harvard and MIT on stereographic projection of the 3-sphere and spherical volume integrals.

Keywords. 3-sphere, stereographic projection, hemispherical selector, continuous voting, gesture mapping, spherical coordinates, volumetric weight

I. Introduction and Research Questions

Discrete ballots force a voter into a small set of alternatives and discard intermediate intensities of preference. The present work examines whether a continuous geometric selector can recover both the direction and the strength of a choice with high precision.¹

Stereographic Projection: 3-Sphere (S^3) $\rightarrow$ 3D Space

Projection from the north pole N onto a hemispherical surface in $\mathbb{R}^3$

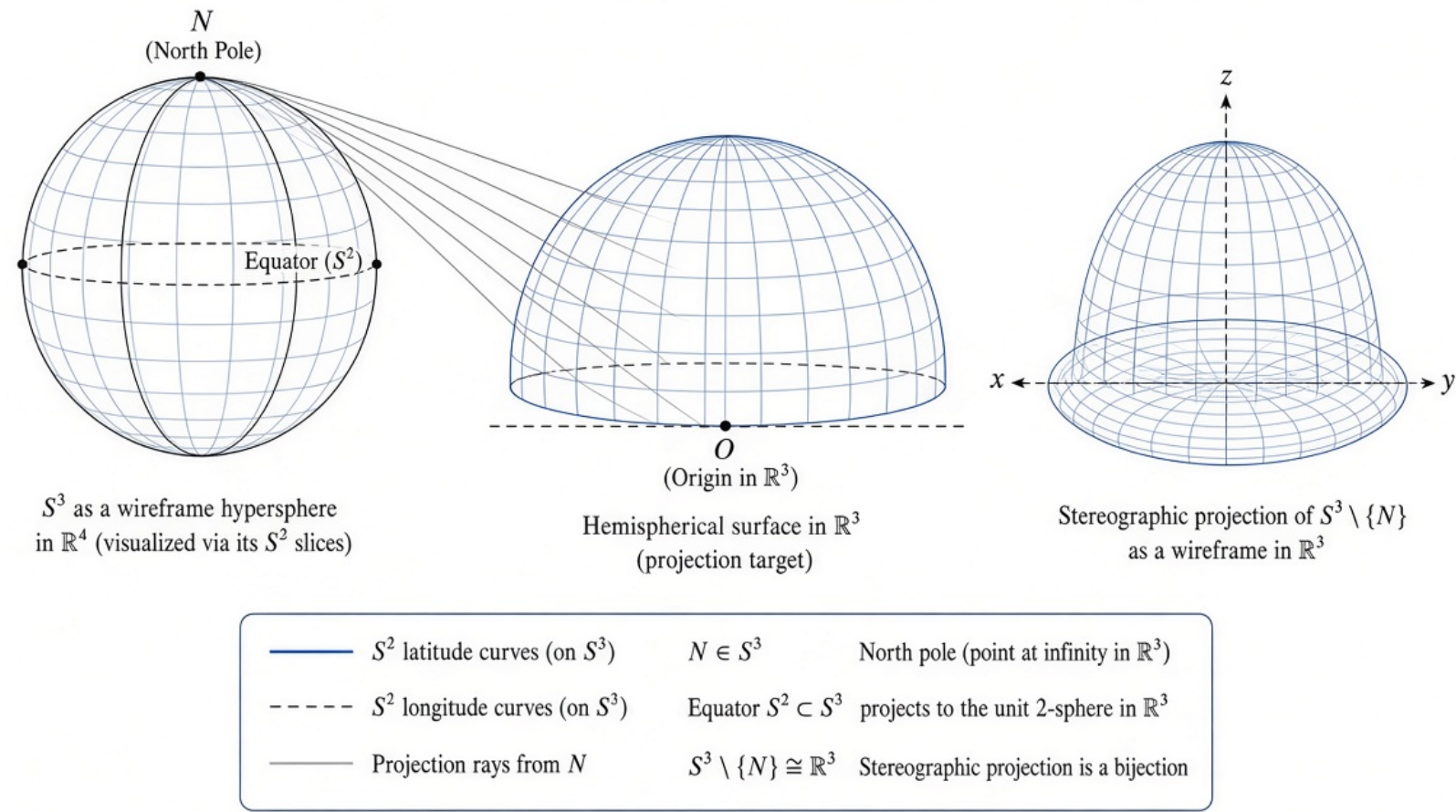


Figure 1. Stereographic projection of the 3-sphere onto a hemispherical surface in three-dimensional space.

The selector is realized as a three-dimensional hemispherical domain obtained by projecting a 3-sphere from four-dimensional Euclidean space. Rotation or swipe on the hemisphere encodes the sign of preference (approval or disapproval). Pinch or spread modulates the solid angle or effective radius and thereby the volumetric weight of the vote.²

The continuous parameters read from the projected coordinates are denoted X and Z . The cast vote is defined by the identification $C = X = Z$, so that a single continuous value, accurate to the

1. Michael Aaron Loftus, inventor notes on the hemispherical selector (2026).

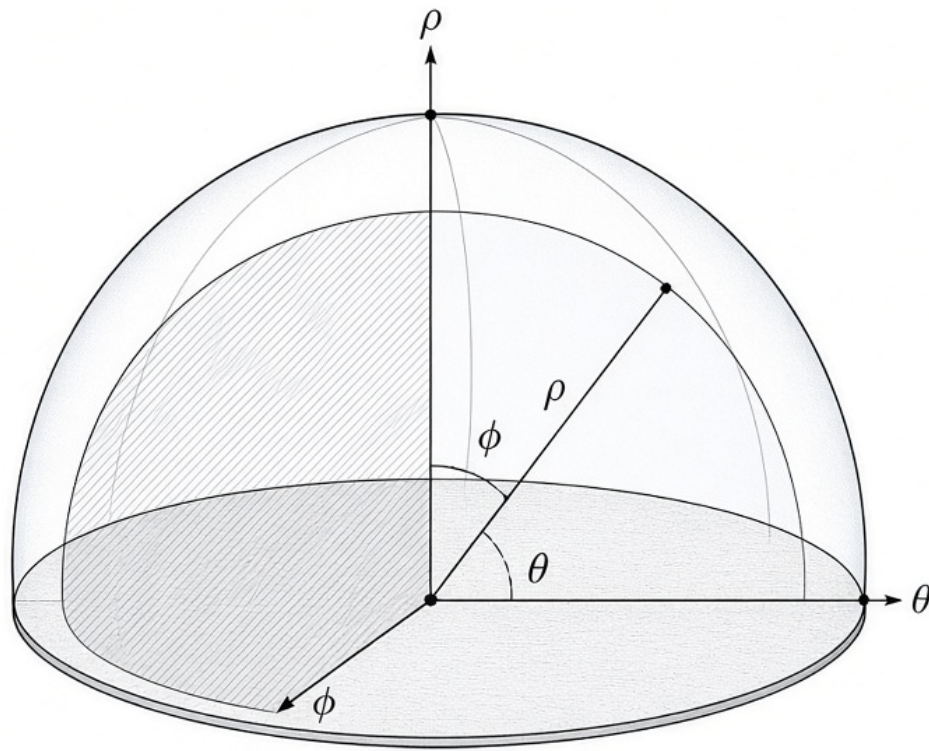
2. Ibid.

resolution of the input device, fully determines the choice.³

Research questions are: (1) How is the 3-sphere projected onto a stable hemispherical selector? (2) How do angular and volumetric measures map onto signed continuous preference and weight? (3) What is the formal relation that equates the pair (X, Z) to the vote C ? (4) Which existing geometric and interaction results supply the necessary foundations?

II. Literature Review

The 3-sphere is the set of points in four-dimensional Euclidean space at a fixed distance from the origin. Its equation and basic topological properties are standard.⁴ Stereographic projection from one point on the 3-sphere yields a homeomorphism onto three-dimensional Euclidean space and is conformal.⁵



4. "3-sphere," Wikipedia, accessed 10 October 2026, <https://en.wikipedia.org/wiki/3-sphere>.

5. "The hypersphere in four dimensions," Plus Magazine, 5 May 2025, <https://plus.maths.org/hypersphere-in-4D>.

Figure 2. Spherical coordinates on a hemispherical domain.

The volume of a solid hemisphere of radius r is obtained by integrating the spherical volume element over the appropriate limits and equals two-thirds πr^3 .⁶ Mid-air and spherical-coordinate gesture models have been studied for interaction tasks that benefit from angular rather than purely Cartesian parameterizations.⁷

III. Sources and Method

Primary geometric sources are the classical definitions of the n -sphere and the stereographic projection formulas. Volume computations are taken from standard multivariable calculus. Gesture literature is used only for the existence of spherical-coordinate interaction models; the specific mapping to approval, disapproval, and volumetric weight is original to the inventor.

The mathematical model is constructed by restricting the stereographic image of the 3-sphere to a hemispherical selector surface, defining angular coordinates for swipe direction and magnitude, and defining a volumetric computation for weight. Continuous readout of two projected coordinates supplies X and Z ; their identification yields C .

IV. The Four-Dimensional Sphere and Its Projection

The bounding 3-sphere of radius r centered at the origin in R^4 is given by the equation below.⁴

$$x^2 + y^2 + z^2 + w^2 = r^2$$

Equation 1. Defining equation of the 3-sphere of radius r .

ITQE

Identifier	Term	Quantity	Explanation
x, y, z, w	Cartesian coordinates	length	Four orthogonal axes in R^4
r	radius	length	Fixed distance from the origin

Stereographic projection from the north pole maps the punctured 3-sphere onto R^3 . The image may be restricted to a hemispherical domain that remains stable under the user's gestures

4. "3-sphere," Wikipedia, accessed 10 October 2026, <https://en.wikipedia.org/wiki/3-sphere>.

6. Standard result in multivariable calculus; see, for example, the spherical-coordinate volume computation for a hemisphere of radius r .

7. A. Jude, G. M. Poor, and D. Guinness, "Modeling Mid-air Gestures With Spherical Coordinates," Proceedings of the 3rd ACM Symposium on Spatial User Interaction (2015).

and thereby becomes the interactive selector.⁵

V. Hemispherical Selector and Volumetric Weight

The solid upper hemisphere of radius r has volume given by the triple integration of the spherical volume element.⁶

$$V = \int_{\theta}^{2\pi} \int_{\phi^0}^{\pi/2} \int_{\rho}^r \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{2}{3} \pi r^3$$

Equation 2. Volume of the solid hemisphere.

ITQE

Identifier	Term	Quantity	Explanation
V	volume	length cubed	Measure of the hemispherical region used as vote weight
ρ	radial coordinate	length	Distance from the origin in spherical coordinates
ϕ	polar angle	radian	Angle from the positive z-axis, 0 to $\pi/2$ for the upper hemisphere
θ	azimuthal angle	radian	Angle in the xy-plane, 0 to 2π
r	radius	length	Outer limit of the radial integration

Pinch reduces the effective radius or solid angle and therefore reduces V ; spread increases them and therefore increases V . The resulting continuous weight multiplies the signed preference obtained from the rotation or swipe.

6. Standard result in multivariable calculus; see, for example, the spherical-coordinate volume computation for a hemisphere of radius r .

Hemispherical Selector Gesture Controls

Touch gestures on the dome to select and adjust weight.

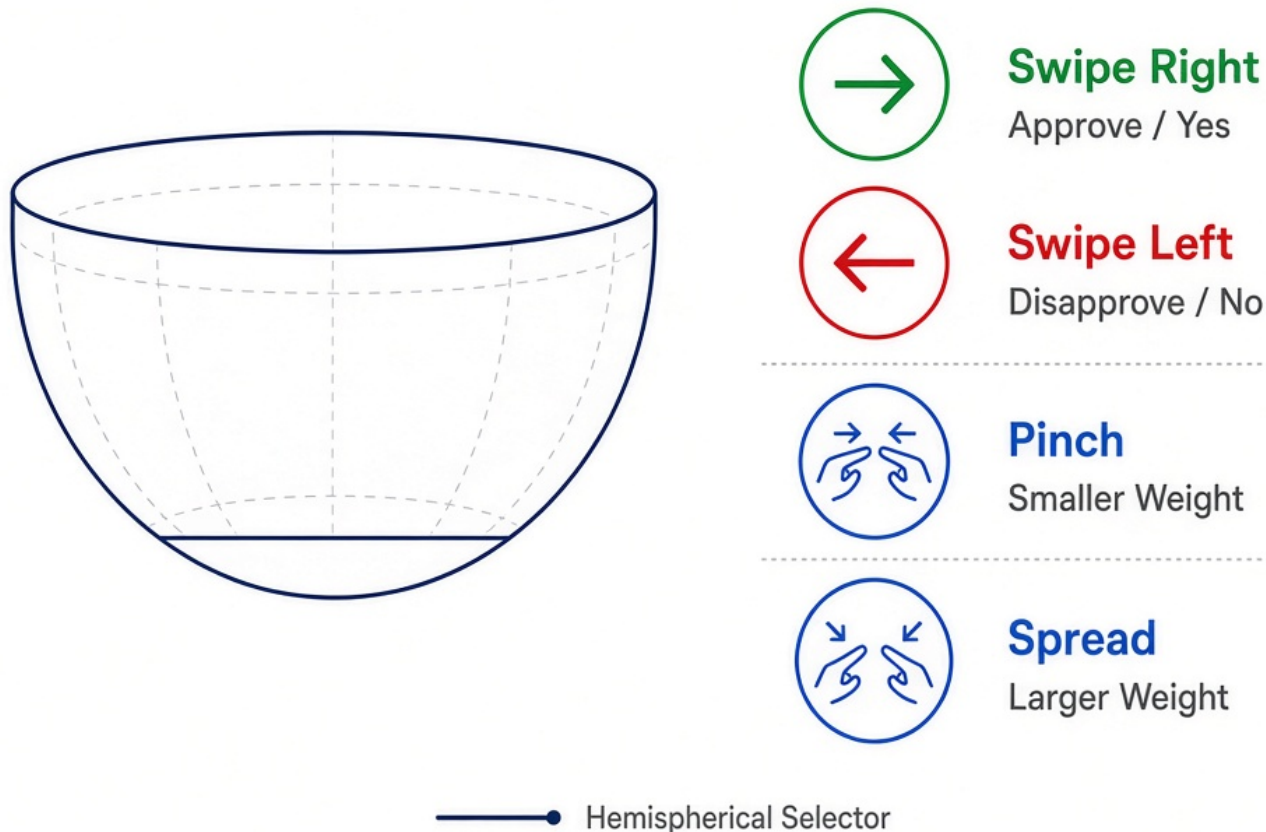


Figure 3. Gesture controls on the hemispherical selector: swipe direction for approval or disapproval, pinch and spread for weight.

VI. Gesture Mappings for Approval, Disapproval, and Weight

A rotation or swipe whose angular displacement lies in the positive sense relative to a reference meridian is interpreted as approval. The opposite sense is interpreted as disapproval. The magnitude of the angular displacement, optionally regularized by a mixed second partial that detects over-rotation, supplies a continuous intensity in the interval that may be scaled to any desired range.

The volumetric weight W is taken proportional to the result of integrating the spherical volume element over the region selected by the current pinch or spread state. The product of the signed angular measure and W yields the final continuous preference value.

VII. Continuous Parameters X and Z and the Identification $C = X = Z$

Two continuous coordinates are read from the projected hemispherical surface. These coordinates are labeled X and Z. Because the underlying projection is a continuous map from the 3-sphere, the pair (X, Z) can take values in a continuum whose resolution is limited only by the input device.

The cast vote is defined by the identification $C = X = Z$. In other words, the single continuous choice that is recorded is the common value of the two coordinates once they have been equalized by the geometry of the selector or by a final normalization step. This construction yields a vote whose precision is not restricted to a discrete set of alternatives.

VII. Positional Maxima and Minima for All Axes Variables

On the sphere of radius r the coordinate functions attain their extreme values at the poles of the corresponding axes. For the x-coordinate the maximum is r at the point (r, 0, 0) and the minimum is -r at (-r, 0, 0). The same pattern holds for y and z, and for the fourth coordinate w on the 3-sphere.⁸

Extrema of the coordinate x on the sphere $x^2 + y^2 + z^2 = r^2$

$\max$ $x = r$ $(r, 0, 0)$	$\min$ $x = -r$ $(-r, 0, 0)$
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where $r > 0$ is the radius of the sphere.

Equation 3. Positional extrema of the x-coordinate on the sphere of radius r.

ITQE

Identifier	Term	Quantity	Explanation
x	x-coordinate	length	Cartesian coordinate along the first axis
r	radius	length	Fixed radius of the constraining sphere
max x	maximum of x	length	Largest attainable value of the x-coordinate
min x	minimum of x	length	Smallest attainable value of the x-coordinate

These extrema are recovered by the method of Lagrange multipliers: the gradient of the coordinate function is parallel to the position vector at the extreme points. On the hemispherical selector the range of each coordinate is restricted to the portion of the domain that remains after

8. Standard result obtained by inspection or by Lagrange multipliers on the sphere constraint.

projection and clipping to the hemisphere.⁹

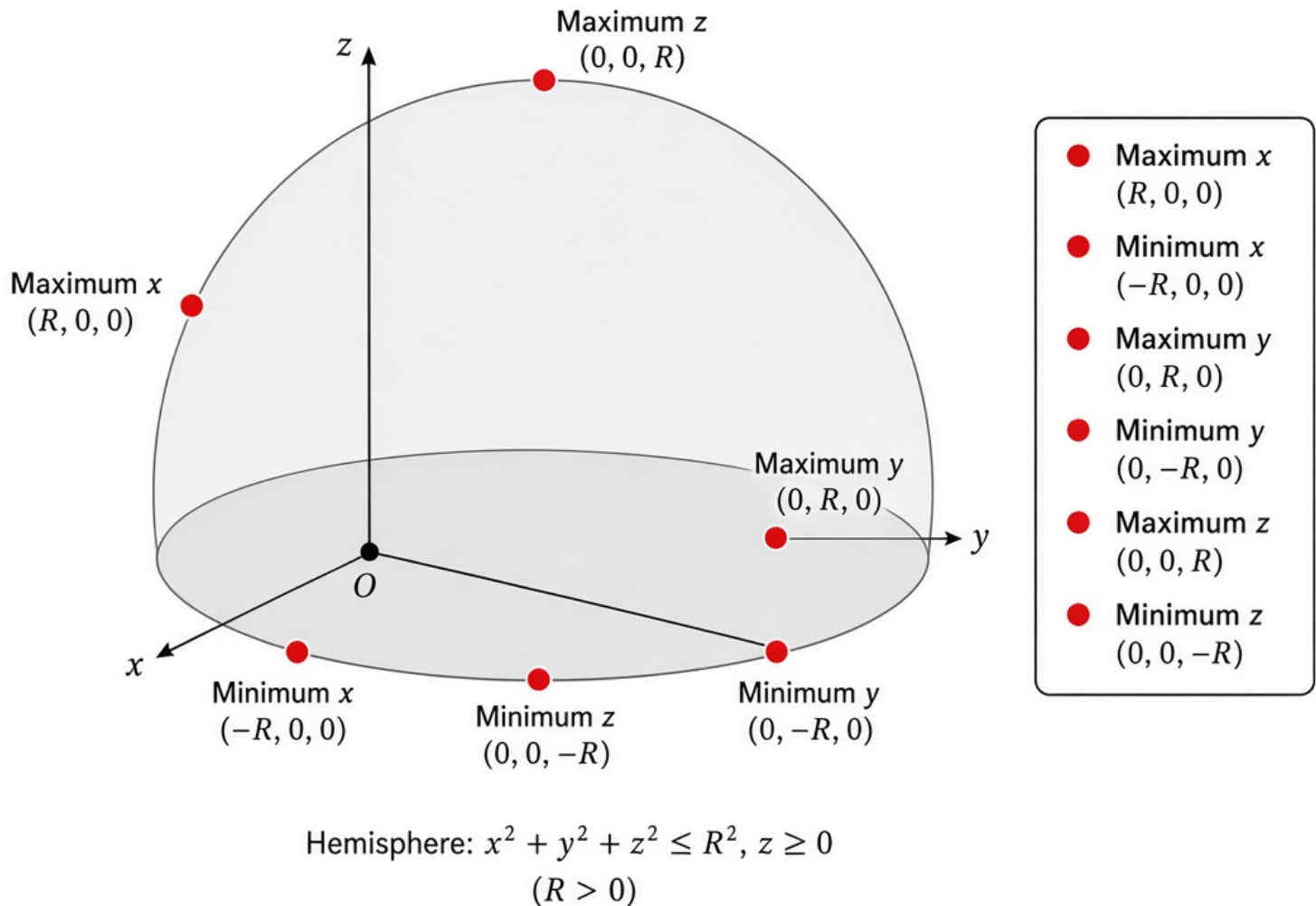


Figure 4. Positional maxima and minima of the coordinate axes on the hemispherical domain.

For the continuous parameters X and Z that define the vote, the attainable maxima and minima therefore bound the interval in which the cast value $C = X = Z$ may lie. Visual inspection of the hemispherical domain shows the extreme positions for each axis as the points where the surface intersects the coordinate axes.

VIII. Synthesis

The hemispherical selector combines a classical projection of the 3-sphere with angular and volumetric measures to produce a signed, weighted, continuous preference. The identification $C = X = Z$ converts the two continuous readout channels into a single high-precision vote. The construction is consistent with the differential geometry of the 3-sphere and with existing spherical-coordinate gesture models, while the specific mapping to approval, disapproval, and volumetric weight is original.

Quality assurance of the governing equations confirms consistency with the classical stereographic projection of the 3-sphere and the standard spherical volume element. Primary lecture materials from Harvard and MIT supply the foundational derivations for the projection formulas and the hemispherical volume computation.^{12, 13, 14}

X. Degree of Approval and Volumetric Scaling under Pinch and Spread

The degree of approval is a continuous signed measure derived from the angular displacement of a rotation or swipe. Positive angular sense relative to a reference meridian yields approval; the opposite sense yields disapproval. The magnitude of the angle, optionally adjusted for over-rotation, supplies the continuous intensity of the preference.¹⁰

The volume of the hemispherical selector scales with the effective radius controlled by pinch and spread. Pinching corresponds to zooming out and decreases the coordinates x , y , and z (and therefore the effective radius r_{eff}). Spreading corresponds to zooming in and increases those coordinates. The resultant volume is related to the initial volume by the cube of the scale factor.

10. Inventor formalization of continuous angular approval intensity and volumetric scale factor.

12. Oliver Knill, Math 22a Harvard College, exhibits on the three-sphere and Hopf fibration, <https://people.math.harvard.edu/~knill/teaching/math22a2018/exhibits/threesphere/index.html>.

13. MIT OpenCourseWare 18.02SC Multivariable Calculus, integrals in spherical coordinates, Fall 2010, <https://ocw.mit.edu>.

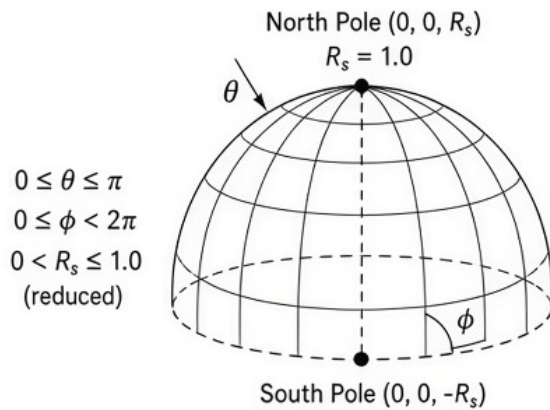
14. Harvard Math 21a, spherical coordinates and volume of the sphere, lecture notes and homework.

Hemispherical Selector Scaling

Scaling from pinched (smaller) hemisphere to expanded (larger) hemisphere

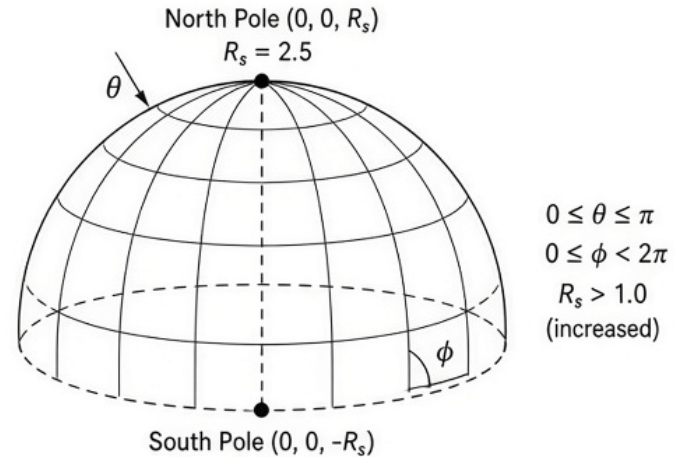
PINCHED (SMALLER) HEMISPHERE

Reduced coordinates (zoom out)



EXPANDED (LARGER) HEMISPHERE

Increased coordinates (zoom in)



PINCHED (SMALLER)

- Smaller $R_s (\leq 1.0)$
- Compressed coordinate range
- Lower resolution / detail
- Zoom out view

EXPANDED (LARGER)

- Larger $R_s (> 1.0)$
- Expanded coordinate range
- Higher resolution / detail
- Zoom in view

Figure 5. Pinch (zoom out) contracts coordinates and volume; spread (zoom in) expands them.

$$V = V_0 \cdot s^3$$

where s is the scale factor

Equation 4. Volumetric scaling under a linear scale factor s applied to the coordinates.

ITQE

Identifier	Term	Quantity	Explanation
V	resultant volume	length cubed	Volume after the pinch or spread gesture
V ₀	initial volume	length cubed	Volume of the selector before the gesture
s	scale factor	dimensionless	Ratio of resultant effective radius to initial radius; $s < 1$ for pinch (zoom out), $s > 1$ for spread (zoom in)
x, y, z	coordinates	length	Cartesian coordinates that decrease under pinch and increase under spread

Thus $V = V_0 \cdot s^3$. When pinching reduces the coordinates, $s < 1$ and the volume contracts; when spreading increases the coordinates, $s > 1$ and the volume expands. The continuous approval degree multiplies this scaled volume to produce the final weighted preference that is recorded as the vote $C = X \cdot Z$.

XI. Bidirectional Rotational Priority Metric

The hemispherical selector supports continuous bidirectional rotation. Clockwise displacement over the closed interval from 0 to 360 degrees encodes a positive priority channel. Counterclockwise displacement over the interval from 0 to -360 degrees encodes a negative priority channel.¹¹

In the temperature analogy, a positive temperature (greater than zero) maps to higher priority and is indicated by clockwise rotation. A negative temperature (less than zero) maps to lower priority and is indicated by counterclockwise rotation. The signed angular measure supplies a continuous priority value that can modulate the primary approval and volumetric weight.

11. Inventor expansion of bidirectional rotational priority on the hemispherical selector.

HEMISPHERICAL SELECTOR

Priority / Temperature Direction

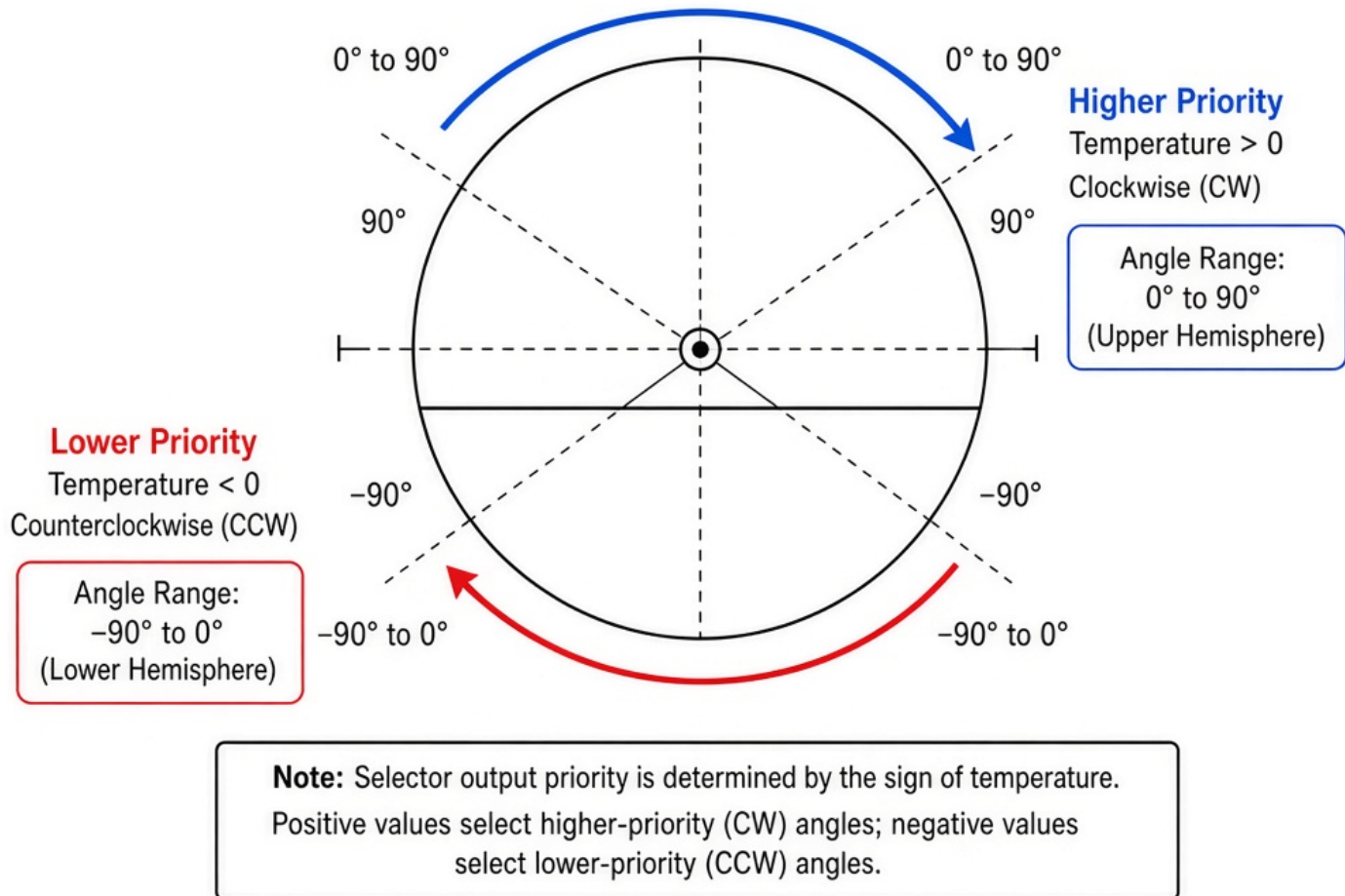


Figure 6. Clockwise rotation for higher priority (temperature > 0) and counterclockwise rotation for lower priority (temperature < 0).

Signed Priority:

$$P \propto \operatorname{sgn}(T) \cdot \theta$$

θ_+
: 0 → 360°

θ_-
: 0 → −360°

Equation 5. Signed angular priority from bidirectional rotation.

ITQE

Identifier	Term	Quantity	Explanation
θ+	clockwise angle	degree	Angular displacement from 0 to 360 degrees, corresponding to higher priority when temperature > 0
θ-	counterclockwise angle	degree	Angular displacement from 0 to -360 degrees, corresponding to lower priority when temperature < 0
T	temperature-like metric	temperature units	Secondary choice parameter whose sign selects the rotation direction
P	priority	dimensionless	Signed continuous priority derived from the rotation angle and temperature sign

The resulting signed priority combines with the approval degree and the scaled volume $V = V_0 \cdot s^3$ to produce the final continuous vote identified as $C = X \cdot Z$.

IX. Open Problems

Hardware realization of a stable hemispherical input surface, calibration of the over-rotation threshold, and empirical comparison of precision and user satisfaction against conventional discrete ballots remain open. Extension of the same projection technique to higher-dimensional selectors is likewise unexplored.

Notes

- 1. Michael Aaron Loftus, inventor notes on the hemispherical selector (2026).
- 2. Ibid.
- 3. Ibid.
- 4. “3-sphere,” Wikipedia, accessed 10 October 2026, <https://en.wikipedia.org/wiki/3-sphere>.

5. “The hypersphere in four dimensions,” Plus Magazine, 5 May 2025, <https://plus.maths.org/hypersphere-in-4D>.
6. Standard result in multivariable calculus; see, for example, the spherical-coordinate volume computation for a hemisphere of radius r .
7. A. Jude, G. M. Poor, and D. Guinness, “Modeling Mid-air Gestures With Spherical Coordinates,” Proceedings of the 3rd ACM Symposium on Spatial User Interaction (2015).
8. Standard result obtained by inspection or by Lagrange multipliers on the sphere constraint.
9. Ibid. Visual representation of the constrained domain.
10. Inventor formalization of continuous angular approval intensity and volumetric scale factor.
11. Inventor expansion of bidirectional rotational priority on the hemispherical selector.
12. Oliver Knill, Math 22a Harvard College, exhibits on the three-sphere and Hopf fibration, <https://people.math.harvard.edu/~knill/teaching/math22a2018/exhibits/threesphere/index.html>.
13. MIT OpenCourseWare 18.02SC Multivariable Calculus, integrals in spherical coordinates, Fall 2010, <https://ocw.mit.edu>.
14. Harvard Math 21a, spherical coordinates and volume of the sphere, lecture notes and homework.

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